DOCUMENT RESUME

ED 231 522 PS 013 712

AUTHOR	Blevins, Belinda; And Others
TITLE	Children's Inferences About Addition and Subtraction Transformations.
PUB DATE	Apr 83
NOTE	15p.; Paper presented at the Biennial Meeting of the Society for Research in Child Development (50th, Detroit, MI, April 21-24, 1983).
PUB TYPE	Reports - Research/Technical (143) -- Speeches/Conference Papers (150)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	Addition; *Arithmetic; *Conservation (Concept); *Developmental Stages; *Elementary School Students; *Preschool Children; Preschool Education; Primary Education; Subtraction
IDENTIFIERS	*Inferential Reasoning; *Numerosity Discrimination

ABSTRACT

A longitudinal study investigated children's ability to infer, from initial and final relative numerosity information, which of four transformations of a stimulus array had occurred: addition, subtraction, expansion, or contraction. It was expected that performance would reflect a sequence of three levels in understanding the effects of addition and subtraction: primitive, qualitative, and quantitative. A group of 68 children from 4.5 to 8 years of age participated. All were twice given the same battery of tasks, with a 1-year interval between assessments. The inference task consisted of parallel sets of primitive, qualitative, and quantitative trials for small number items (from 2 through 4) and large number items (from 7 through 9). On primitive inference trials two equal linear arrays of squares were presented. On qualitative inference trials the arrays differed by 1, and on quantitative inference trials the arrays differed by 2. Arrays were presented, described, transformed, and erased; the child was required to decide which transformation had been performed. In addition/subtraction trials, children were given relative numerosity information, saw a transformation, and made a judgment about the final relative numerosity. At the beginning and end of the battery of tasks, children were given number conservation tasks and scored as passing if they gave adequate explanations for correct judgments on large number trials. All tasks were presented on a color monitor attached to an Apple II computer. The systematic relationships found in this study suggest that it may be useful to focus on identifying general developmental changes occuring across related areas. (RH)

Children's Inferences About Addition and Subtraction Transformations

Belinda Blevins, Robert Campbell, and Robert G. Cooper, Jr.

University of Arkansas at Little Rock, University of Texas at Austin, and Southwest Texas State University

Paper presented at the Society for Research in Child Development in Detroit, April 1983

The research presented here concerns children's reasoning about addition and subtraction. It is focused on reasoning that develops without formal tuition and on reasoning done without the aid of the counting estimator. Our previous work with preschoolers and first and second graders has supported the existence of a three-level sequence in understanding the effects of addition and subtraction (Blevins, Mace, Cooper, Starkey, & Leitner, 1981; Cooper, Campbell, & Blevins, 1982; Cooper, Starkey, Blevins, Goth, & Leitner, 1978). The levels are called primitive, qualitative, and quantitative and are diagrammed in Table 1. At the primitive level childre believe that adding makes more and subtracting makes less. At the qualitative level, they distinguish between more than before and more than another group of objects; however, they do not quantify the difference between the two groups. If there are two unequal groups of objects (with a difference of ≤ 3) they predict that adding to the smaller one (or subtracting from the greater) will make the groups equal. At the quantitative level, children can quantify the difference between the two arrays.

Recently we completed a two year longitudinal study of early mathematical skills in which we investigated children's ability to infer, from initial and final relative numerosity information, which of four transformations had occurred: addition, subtraction, expansion, or contraction. The ability to make such inferences involves applying what is known about both addition and subtraction and number conservation. In order to make the inference, children have to be able to understand the difference between number changing and number maintaining transformations and they have to know exactly what effect each transformation has. Since this inference ability should be influenced by an increasing ability to quantify, we expected performance to reflect the three level sequence already outlined.

Sixty-eight children, from 4½ to 8 years of age, participated in the stydy. All were given the same battery of tasks twice, with a one year interval between assessments. The tasks included several in addition to the inference, number conservation and addition/subtraction tasks, but only data concerning the inference task in relation to addition/subtraction and conservation will be reported. All the tasks were presented on a color monitor which was attached to an Apple II computer.

Inference task. The inference task consisted of parallel sets of primitive, qualitative, and quantitative trials for small number (2-4) and large number (7-9). Since children can subitize small numbers, they can represent the information about those trials in terms of absolute number. On each trial, two linear arrays of squares were presented. On primitive trials, the 2 arrays were equal, on qualitative trials the arrays differed by one, and on quantitative trials the arrays differed by 2. The arrays were lined up in spatial one-to-one correspondence, and were separated by a horizontal white line. The experimenter used one-to-one correspondence cues to demonstrate the relative numerosity of the arrays. If one of the arrays had more squares, the experimenter stated how much more it had. Then both arrays were screened with large colored rectangles. The experimenter stated that one of the arrays would be changed and indicated which one. The transformation could be one of four: expand, contract, add 1, or subtract 1. Finally, the arrays were unscreened and the experimenter described their final relative numerosity, including how much more when relevent. The experimenter reminded the child which array had been transformed, and restated the initial relative numerosity. The child had to decide which transformation had been performed. All children were asked not to count.

Because the inference task is complex and its memory demands are extensive, care was taken that all of the children knew the four transformations. Before starting the small-number assessment, children were shown four practice trials in which the transformations were not screened. The experimenter named the transformations and made sure the child could recite all 4 before going on to the small-number assessment.

As the inference task was piloted in the first year of the study, only data from the second year will be reported. Children were classified as primitive, qualitative, or quantitative for small number according to the highest level for which they answered 3 out of 4 trials correctly. If their response did not meet this criterion, no level was assigned. Similarly, each child was classified as primitive, qualitative, or quantitative for large number. Most children were classified as belonging to one of our levels (see Table 2). As expected, children did better on small number problems than large number problems -- the majority of children were classified at the quantitative level for small number. Reaching the quantitative level for small number appears to be a prerequisite for being quantitative for large number since most of the children who were quantitative for large number were quantitative for small number (binomial test, $p < .005$), but, children who were quantitative for small number were represented at all the levels for large number.

Most nursery school children were at the primitive level or no level for large number. Kindergarten and first grade children were distributed across all levels. Only in second grade were children predominantly quantitative for large number. Those who were scored as "no level" did not make random errors. They failed to distinguish number transformations from length transformations, confusing addition with expansion and subtraction with contraction ($\chi^2 = 16.44$, df = 1, $p < .01$). An analysis of the errors of all the children revealed that the majority of errors involved the same type of confusion ($\chi^2 = 20.02$, df = 1, $p < .01$).

Addition/Subtraction and Inference. The addition/subtraction assessment consisted of primitive, qualitative, and quantitative trials which are outlined in Table 1. Children were given initial relative numerosity information which meant that they were told whether both arrays had the same number, or how many more the larger array had. Then they saw a transformation, and were asked to make a judgment about the final relative numerosity. Only large number addition/subtraction will be considered as the children performed almost perfectly on small number addition/subtraction. If children were classified as being large quantitative on addition/subtraction they were likely to be classified as passing small quantitative inferences. When inference performance for large number was compared to addition/subtraction performance for large number children were nearly always at the same level or lower for inference than for addition/subtraction ($\chi^2 = 28.27$, df = 1, $p < .01$). There appeared to be a lag of a year or more between attaining the quantitative level on the addition/subtraction task and reaching the same level on the large number inference task. Children can predict final relative numerosity, knowing the addition/subtraction transformation and the initial relative numerosity, well before they can use initial and final relative numerosity to infer the exact transformation.

In both the inference task and the addition/subtraction task, small number trials were easier than large number trials. This could be due to the fact that children can form a representation based on absolute numerosity in the small number trials. One difference between the inference task and the addition/subtraction task is that in the addition/subtraction task explicit knowledge of the numerosity is sufficient to solve the task, but an algebraic understanding is involved in solving the inference task. This same distinction may account for the more general finding that small number tasks are easier than large number tasks. Certainly an explicit knowledge of numerosity can be used to solve large number addition/subtraction tasks, but children were asked not to count. In this case, an algebraic representation is needed to solve the task.

The same levels of understanding addition/subtraction characterize performance on the inference task and the addition/subtraction task. One commonality between the tasks is that they both involved reasoning about quantity, so the development of quantification skills may underlie the similar developmental patterns. Research by Goth (1981) indicates that the same 3 levels of understanding are found in children's reasoning about quantities of length and amount. This suggests that the development of quantification skills as identified by our model may have general relevance to children's reasoning about quantity.

<u>Conservation and Inference</u>. Children were given two number conservation assessments, one at the beginning of the battery of tasks and one at the end. In the number conservation task, children were asked for judgments and explanations and they were scored as passing if they gave adequate explanations for correct judgments on large number trials. Passing qualitative or quantitative inference for large number was associated with passing conservation ($\chi^2 = 13.38$, df = 1, $p < .01$), although some conservers did poorly on large number inference. Nonconservers scored at the primitive level or no level for large number,

indicating their failure to distinguish clearly between number and length transformations. Children need to conserve (i.e., understand that addition/subtraction changes number and that length transformations do not) if they are to do well on the inference task. But conservation is not sufficient to solve the task. A quantitative understanding of addition/subtraction is also needed.

Correlations Between Tasks. The final analysis involved computing correlations between all the tasks. These correlations are listed in Table 4. All the tasks are positively correlated.

Conclusion. The inference task we have developed assesses children's ability to infer a transformation from initial and final information about relative numerosity. This ability marks an important application of the child's knowledge of addition/subtraction. The application of this knowledge involves a more explicit understanding of addition/subtraction principles based on the ability to represent information about number algebraically. Correspondingly, we have found that performance on the inference task is lawfully relation to addition/subtraction and conservation performance. These systematic relationships suggest that it may be useful to focus on identifying general developmental changes occurring across related areas. Our three-stage sequence is a step in this direction.

References

Blevins, B., Mace, P., Cooper, R., Starkey, P., and Leitner, E. What do children know about addition and subtraction? Paper presented to the meeting of the Society for Research in Child Development, Boston, 1981.

Cooper, R., Campbell, R., and Blevins, B. Numerical representation from infancy to middle childhood: What develops? Paper presented at the NATO Conference Acquisition of Symbolic Skills, University of Keele, Keele, England, 1982.

Cooper, R., Starkey, P., Blevins, B., Goth, P., and Leitner, E. Number development: Addition and subtraction. Paper presented at the Jean Piaget Society Meetings, Philadelphia, 1978.

Goth, P. The development of addition-subtraction knowledge and its relation to conservation in young elementary school children. (Doctoral dissertation, University of Texas, 1980). Dissertation Abstracts International, 1981, 41, 2791B. (University Microfilms No. 8100907).

Table 1

Trial Type and Pattern of Performance for Addition/Subtraction Experiments

Performance Level	Trial Type					
	n +1 n+1 +1 n+2 -------> -------> n n n OR n -1 n-1 -1 n-2 -------> -------> n n n		n-1 +1 n +1 n+1 -------> -------> n n n OR n+1 -1 n -1 n-1 -------> -------> n n n		n-2 +1 n-1 +1 n -------> -------> n n n OR n+2 -1 n+1 -1 n -------> -------> n n n	
Primitive	correct	correct	incorrect	incorrect	incorrect	incorrect
Qualitative	correct	correct	correct	correct	incorrect	incorrect
Quantitative	correct	correct	correct	correct	correct	correct

Table 2

Distribution of Performance Levels by Grade

on the Inference Task

	Level for Small Number				Level for Large Number				
Grade	None	Prim	Qual	Quan	None	Prim	Qual	Quan	
N	1	3	1	4	3	4	1	1	9
K	1	1	4	9	6	1	3	5	15
1	0	2	9	15	6	6	5	9	26
2	0	0	2	15	0	1	4	12	17
	2	6	16	44	15	12	13	27	

Table 3

Distribution of Performance Levels on Inference Task by Performance on the Addition/Subtraction Task

Addition/Subtraction Level	Level for Small Number				Level for Large Number			
	None	Prim	Qual	Quan	None	Prim	Qual	Quan
Large Prim	0	1	1	0	2	0	0	0
Large Qual	2	1	6	8	6	5	3	3
Large Quan	0	4	10	36	8	7	10	24

Table 4

Correlations Between Inference Task, Addition/Subtraction Task, and Conservation Task

	Small-Number Inference	Large-Number Inference	Large Addition/Subtraction	Conservation Assessment 1	Conservation Assessment 2
Small-Number Inference	----	.42** (70)	.34* (69)	.58** (71)	.51** (61)
Large-Number Inference	----	----	.36** (68)	.55** (70)	.43** (61)
Large Addition/Subtraction	----	----	----	.50** (70)	.47** (60)
Conservation Assessment 1	----	----	----	----	.59** (62)
Conservation Assessment 2	----	----	----	----	----

* $p < .01$

** $p < .001$